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# Fluidization Engineering

SECOND EDITION

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***An Imprint of Elsevier***

Boston London Singapore Sydney Toronto Wellington



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#### Library of Congress Cataloging-in-Publication Data

Kunii, Daizo, 1923-  
Fluidization engineering / Daizo Kunii, Octave Levenspiel.  
—2nd ed.  
p. cm.—(Butterworth-Heinemann series in chemical engineering)  
ISBN 0-409-90233-0  
1. Fluidization. I. Levenspiel, Octave. II. Title.  
III. Series.  
TP156.F65K8 1991  
660'.284292—dc20

90-31800

#### British Library Cataloguing in Publication Data

Kunii, Daizo  
Fluidization engineering.  
1. Chemical engineering. Fluidisation  
I. Title II. Levenspiel, Octave  
660.2842  
ISBN 0-409-90233-0

Butterworth-Heinemann  
313 Washington Street  
Newton, MA 02158-1626

10 9 8 7



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## Solid Movement: Mixing, Segregation, and Staging

- Vertical Movement of Solids
- Horizontal Movement of Solids
- Segregation of Particles
- Large Solids in Beds of Smaller Particles
- Staging of Fluidized Beds
- Leakage of Solids through Distributor Plates

As shown in Chap. 6 the channeling of the rising gas bubbles causes the gross circulation of solids in a fluidized bed, while the small-scale intermixing of particles occurs mainly within the wakes that accompany the bubbles up the bed. When solids of wide size distribution and/or of different densities are fluidized, the larger or heavier particles tend to settle to the bottom of bed, but this is countered by the solid circulation, mentioned above. At several multiples of  $u_{mf}$  of the largest or heaviest particles, the mixing process dominates. However, as the gas velocity is reduced to and then below  $u_{mf}$  of the largest or heaviest particles, these solids progressively concentrate at the bottom of the bed. Thus mixing and segregation of different solids is apparently an equilibrium process that depends on bed conditions. Since vertical segregation of different solids is absent in high-velocity fluidization typical of fast fluidization or pneumatic transport, this chapter only concerns bubbling and turbulent beds wherein  $u_o$  is close to  $u_{mf}$  of at least some of the bed solids.

The rate of horizontal mixing of solids is also of concern. This is especially so in long shallow beds wherein solids are fed at one end of the bed, react in the bed, and then leave at the other end of the bed.

Overall, there are numerous aspects to the mixing and movement of solids in fluidized beds. In this chapter we consider

- Vertical mixing and segregation of solids
- Horizontal mixing and dispersion of solids
- Mixing-segregation equilibrium
- Large solids in beds of small particles
- Transfer of solids across horizontal baffle plates and leaking through distributor plates

For the design of a number of physical and chemical processes it is important to understand the mechanism and rates of these opposing phenomena of mixing and segregation and related phenomena. In some situations one may even take



advantage of and deliberately encourage segregation of solids in the development of improved processes.

## Vertical Movement of Solids

In catalytic reactors the large-scale vertical movement of porous particles can carry large amounts of adsorbed reaction components up and down the bed. This type of gas back mixing usually lowers conversion and selectivity. This is one reason why we need to know how much mixing of solids does occur, how to model this mathematically for predictive purposes, and what means are available to depress this movement.

As mentioned in Chap. 2, one often finds vertical or horizontal tubes fitted in catalytic reactors. These are placed there for various reasons: for temperature control, to reduce gulf circulation of solids, to reduce bubble size, to increase the emulsion voidage and thereby increase the overall residence time of reactant gas in the bed. All this raises the conversion of reactant gas and improves the selectivity of the desired product. For this reason various groups have also studied the movement of solids in beds with internals.

## Experimental Findings

A variety of techniques have been used to study the vertical movement of solids, for example:

- Following the paths of individual tagged particles for long periods of time as they move about the bed.
- Measuring the extent of intermixing of two kinds of solids, originally located one above the other in the bed.
- Measuring the vertical spread of a thin horizontal slice of tracer solid.
- Finding the residence time distribution of the flowing stream in a bed with a throughflow of solids, using a variety of tracer techniques, such as step or pulse injection.
- Measuring the axial heat flow in a bed with a heated top section and cooled bottom section. This technique assumes that heat transport is caused solely by the movement of solids.

Surveys of the results of these many experimental studies are given by Kunii and Levenspiel [1], Potter [2], and van Deemter [3] in 1969, 1971, and 1985, respectively. These findings are most often reported in terms of the vertical dispersion coefficient  $D_{sv}$ . We briefly summarize them.

**Beds without Internals.** Figure 1(a) shows that the vertical mixing rate in rather small beds is directly related to the gas velocity by

$$D_{sv} = 0.06 + 0.1u_o, \quad [\text{m}^2/\text{s}] \quad (1)$$

and Fig. 1(b) shows that the vertical mixing of solids is more rapid in large-diameter beds than in smaller beds, the relationship being given by

$$D_{sv} = 0.30d_t^{0.65}, \quad [\text{m}^2/\text{s}] \quad (2)$$

In fact, in large vigorously bubbling fluidized beds of fine solids, the vertical movement of solids is very rapid. As an example, May [4] found that a slug of



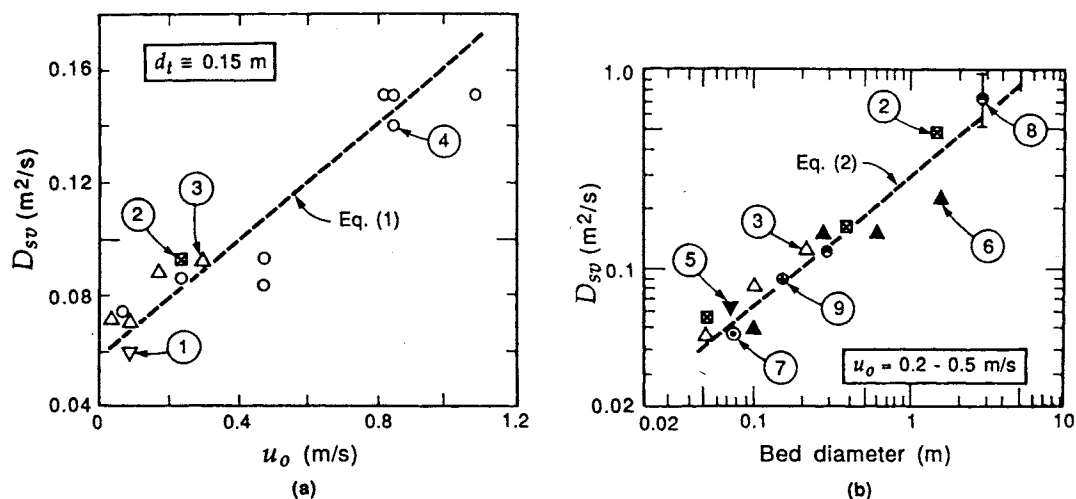


FIGURE 1

Vertical dispersion of solids in fine particle fluidized beds; from Avidan and Yerushalmi [5]; data from de Vries et al. [6] are added, and correlation line is modified somewhat from the original. ① Stermerding (Reman, 1955) [7], ② May (1959) [4], ③ Thiel and Potter (1978) [8], ④ Avidan and Yerushalmi (1985) [5], ⑤ Lewis et al. (1962) [9], ⑥ de Groot (1967) [10], ⑦ Miyauchi et al. (1968) [11], ⑧ de Vries et al. (1972) [6], ⑨ Avidan (1985) [5].

tracer introduced at one location in a large bed ( $d_t = 1.52$  m,  $L_f = 9.75$  m) of FCC catalyst became uniformly distributed throughout the bed in about 1 min.

Miyauchi et al. [12] studied the vertical mixing of solids in vigorously fluidized ( $u_o > 10$  cm/s) beds of fine Geldart A solids. Recall from Chap. 3 that in such beds bubbles quickly reach a small limiting size. They found that the mixing data could reasonably be represented by the dispersion model with

$$D_{sv} = 12u_o^{1/2}d_t^{0.9}, \quad [\text{cm}^2/\text{s}] \quad (3)$$

Example 1 compares values from this equation with the experimental values reported by de Groot [10].

Unfortunately, the dispersion model does not always well represent the vertical movement of solids. For example, May [4] found that for Geldart A solids the model well fitted the solid movement in his bed of aspect ratio  $L_f/d_t = 9.1$  m/0.38 m = 24, but was inadequate for this bed of aspect ratio  $L_f/d_t = 9.75$  m/1.52 m = 6.4.

Similarly, Avidan and Yerushalmi [5] found that the dispersion model well represented the mixing during turbulent fluidization where the bed looked close to homogeneous, but fitted the data poorly when the bed was in the bubbling regime.

To summarize, one may expect the dispersion model to reasonably represent the vertical mixing in tall beds in which rather small-scale mixing is taking place. This is characteristic of fine particle (Geldart A) systems with only mild gulf streaming. We would not expect it to satisfactorily represent shallow beds, beds with strong solid circulation, or beds with nonuniformly distributed internals.

Where the dispersion model does not fit well (gently bubbling and in not



very deep beds), the countercurrent mixing model often is used. This model views the solids moving in two streams, one rising and the other descending, with a crossflow or interchange between streams. This model closely matches the K-L model of Chap. 6 and is taken up in the next section.

Although the dispersion model does not reasonably represent the movement of solids in certain conditions, the results of experiments are invariably forced into this form and are reported in terms of a dispersion coefficient.

**Beds with Internals.** So far we have presented the findings on solid movement in beds free of internals. However, the presence of internals in a bed strongly hinders this movement. For example, Chen et al. [13] reported the following velocities of solids in the core of the upper vortex (refer to Fig. 6.3(d) of their experimental bed containing horizontal tubes:

	No tube bundle in bed	Sparse bundle	Dense bundle
Average upward solid velocity (m/s) $\left\{ \begin{array}{l} \text{at } u_o/u_{mf} = 4 \\ \text{at } u_o/u_{mf} = 6 \end{array} \right.$	0.19 0.26	0.09 0.15	0.01 0.02

Admittedly the bed was rather small ( $d_t = 0.19$  m,  $L_f \approx d_t$ ); however, these findings should indicate the general effect of bed internals.

In experiments with larger Geldart B solids in a  $1.2 \times 1.2$  m bed that contained internals, Sitnai [14] noted that if a tube bank was located away from the wall of the bed, then solids would slide down in a thick stream along the wall surface, thereby generating a severe solid circulation pattern. So, to achieve a uniform solid movement, one should take this into account.

As an extreme of bed internals, Claus et al. [15] reported on the behavior of a fluidized bed ( $d_t = 9.2$  cm,  $L_m = 9.2$  m) packed with 2-cm wire screen Raschig rings. With Geldart B solids they found remarkably uniform fluidization with small bubbles throughout the bed. However, with fine Geldart A solids, fluidization was unsatisfactory, with considerable agglomeration of fine solids on the packing.

We next discuss the various models that have been used to interpret the experimental findings on the vertical movement of solids.

### Dispersion Model

The *dispersion model* is a diffusion-type model represented by the differential equation

$$\frac{\partial C_s}{\partial t} = D_{sv} \frac{\partial^2 C_s}{\partial z^2} \quad (4)$$

where  $C_s$  is the concentration of tagged particles at position  $z$  at time  $t$ , and  $D_{sv}$  is the vertical dispersion coefficient averaged over the entire cross section of the bed.

The solution of Eq. (4) may take several forms. For a step input of tracer introduced into the stream of solids entering the bottom of a fluidized bed and leaving at the top, or vice versa, the use of the appropriate boundary and initial



conditions gives

$$\frac{C_s(\text{at exit})}{C_s(\text{at } t = \infty)} = f\left(\frac{D_{sv}}{\bar{u}_{s,up} z_f}, \frac{t}{t_s}\right) \quad (5)$$

For a pulse of tracer introduced into a bed with no throughflow of solids,

$$\frac{C_s(t)}{C_s \text{ if well mixed in the bed}} = f\left(\frac{D_{sv} t}{L_f^2}, \frac{z}{L_f}\right) \quad (6)$$

Verloop et al. [16] gave several solutions to Eq. (5) for solids throughflow, and also told where additional solutions may be found. For the batch situation, May [4] gave the solution for one initial condition. Others can be extracted from Carslaw and Jaeger [17].

As understanding of the hydrodynamics of fluidized beds grew, attempts were made to relate the dispersion model to more mechanistic models so that more fundamental measurements could be used for the design of large-scale units. We now look at some of these developments.

### Counterflow Solid Circulation Models

In the bubbling bed models sketched in Fig. 6.12, we see some solids flowing up the bed and others flowing down the bed. This upflow and downflow with an interchange between streams is the basis for various counterflow models that have been proposed to account for the vertical mixing of solids.

The simplest version, introduced by van Deemter [18], divides the solids into two streams: one flowing up at a velocity  $u_{su}$ , the other flowing down at  $u_{sd}$ , with  $f_u$  and  $f_d$  ( $= \text{m}^3 \text{ solids/m}^3 \text{ bed}$ ) being the bed fractions consisting of these streams (see Fig. 2). Consider the movement of some labeled or tagged

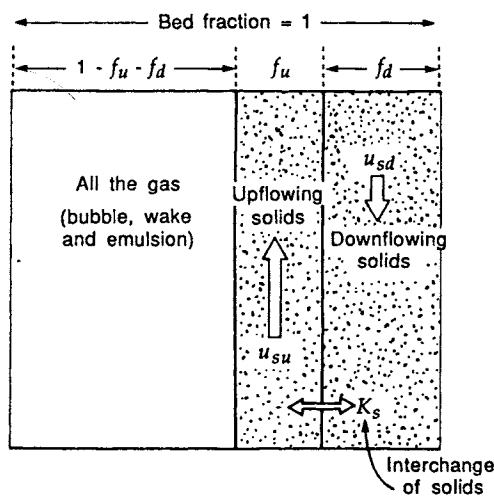


FIGURE 2

Counterflow solid circulation model for the vertical movement of solid in a bubbling fluidized bed.



solids that constitute a fraction  $X_{su}$  and  $X_{sd}$  ( $= \text{m}^3$  tagged solids/ $\text{m}^3$  total solid stream) of the up- and downflowing streams. The differential equation describing the vertical movement of these tagged solids and their interchange is then

$$f_d \frac{\partial C_{sd}}{\partial t} + f_d u_{sd} \frac{\partial C_{sd}}{\partial z} + K_s(C_{sd} - C_{su}) = 0 \quad (7)$$

and

$$f_u \frac{\partial C_{su}}{\partial t} + f_u u_{su} \frac{\partial C_{su}}{\partial z} + K_s(C_{su} - C_{sd}) = 0 \quad (8)$$

where the solids interchange coefficient  $K_s$  ( $\text{m}^3$  tracer/ $\text{m}^3$  bed·s) represents the transfer of tagged solid from one stream to the other.

For a tall enough bed of fine particles and sufficiently large values of elapsed time, van Deemter showed that the changes in concentration of labeled solids could be represented by an effective dispersion coefficient given by

$$D_{sv} = \frac{f_d^2 u_{sd}^2}{K_s(f_d + f_u)} = \frac{f_d^2 u_{sd}^2}{K_s(1 - \delta)(1 - \epsilon_f)} \quad [\text{m}^2/\text{s}] \quad (9)$$

He then applied this relationship to the data on vertical mixing of silica sand ( $u_{mf} = 0.15 \text{ cm/s}$ ) reported by de Groot [10], and found values of  $D_{sv}$  ranging from 0.03 to  $0.23 \text{ m}^2/\text{s}$ .

### Relating the Counterflow to the Dispersion Model

Kunii et al. [19] proposed using the Davidson bubble plus wake as the basis for developing an expression for the interchange coefficient between up- and downflowing solids in beds of fast-rising, hence clouded, bubbles.

Consider the movement of solids around a clouded bubble as shown in Fig. 3. Since the circulation of cloud gas is rapid, Kunii et al. assumed that all the solids from the lower part of the cloud are swept into the wake, mix with the

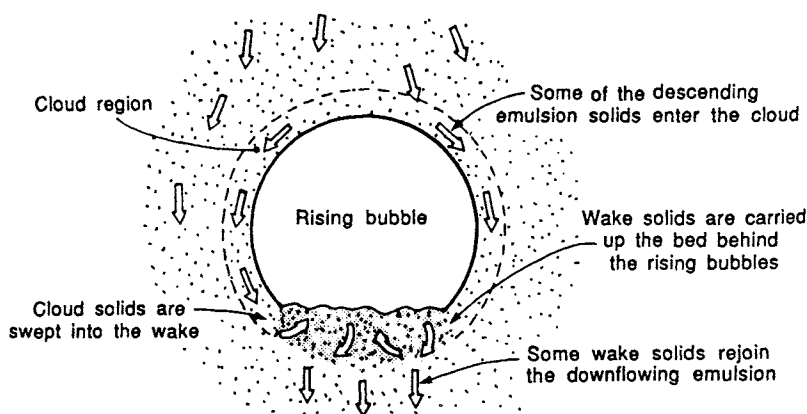


FIGURE 3

Model for the mechanism of interchange of solids between downflowing emulsion solids and upflowing wake solids; from Kunii et al. [19].



solids already there, and eventually leak back into the emulsion. By this process, slowly downflowing emulsion solids are swept into the rising bubble wake and then return to the downflowing emulsion. From this mechanism the interchange coefficient for solids in beds with clouded bubbles is

$$K_s = \frac{(\text{volume of solids transferred from the emulsion to the wake})}{(\text{volume of bubble})(\text{time})}$$

$$\approx \frac{3(1 - \epsilon_{mf})}{(1 - \delta)\epsilon_{mf}} \frac{u_{mf}}{d_b}, \quad [s^{-1}] \quad (10)$$

With a somewhat similar model, Chiba and Kobayashi [20] derived the following expression for the interchange coefficient:

$$K_s = \frac{3}{2} \left( \frac{f_w}{1 + f_w} \right) \frac{u_{mf}}{\epsilon_{mf} d_b} \quad (11)$$

Introducing  $K_s$  from Eq. (10) into Eq. (9) and simplifying leads to the following expression for the vertical dispersion coefficient in terms of measurable bubble and bed properties:

$$D_{sv} \approx \frac{f_w^2 \epsilon_{mf} \delta d_b u_b^2}{3 u_{mf}} \quad (12)$$

Potter [2] developed a similar expression with the term  $1 - \delta$  multiplied on the right-hand side, and with  $u_o$  in place of  $u_b$ .

As mentioned, van Deemter [18] extracted values of  $D_{sv}$  from the experimental results reported by de Groot [10]. Example 1 compares the predictions of Eq. (12) with these reported  $D_{sv}$  values. By estimating appropriate values of the bubble rise velocity, one obtains a good fit. In particular, Eq. (12) predicts that  $D_{sv}$  should be larger for beds of smaller particles. This is consistent with experimental findings.

### Coarse Particle Beds

So far we have only considered the movement of fine solids in tall beds, thus beds with small equilibrium bubbles. When coarse particles are fluidized in beds of aspect ratio of unity, or shallower still, the application of the dispersion model to vertical mixing is not justified unless internals are positioned exactly uniformly across the bed, in which case the spacing of the internals governs the scale of solid mixing.

Such positioning is not really practical; hence one may find a thick stream of solids descending at vessel walls or in the spaces between neighboring tube banks. Sitnai [14] proposed another counterflow model, which accounts for this third phase of solids that descends along these walls. This model is sketched in Fig. 4. On fitting his model to tracer data from a 1.2 m × 1.2 m tube-filled bed, he gave the mean velocity of descending solids as 1.7–1.8 cm/s in the main part of the bed and 4.8–5.8 cm/s along the wall for  $u_o = 0.6$ –0.9 m/s and  $u_{mf} = 0.3$  m/s. Van Deemter [3] surveyed these counterflow solid circulation models.



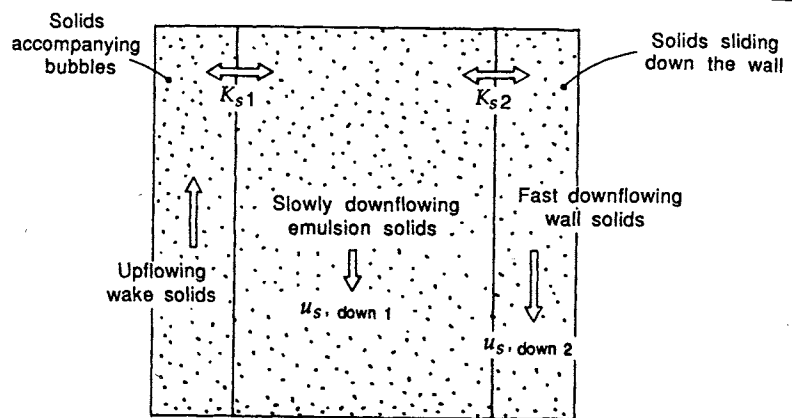


FIGURE 4

Three-region counterflow model for vertical solid movement in large particle shallow beds with internals; from Sitnai [14].

**EXAMPLE 1**

**Vertical  
Movement  
of Solids**

Calculate the vertical dispersion coefficient from Eqs. (3) and (12) and compare these values with the values extracted by van Deemter [18] from the experiments of de Groot [10] in various sized vessels.

**Data**  $u_{mf} = 0.015 \text{ m/s}$ ,  $\varepsilon_{mf} = 0.5$ ,  $u_o = 0.1 \text{ m/s}$ ,  $\delta = 0.2$ ,  $d_b = 0.06 \text{ m}$

$d_t \text{ (m)}$	0.1	0.3	0.6	1.5
$u_b \text{ (m/s)}$	0.40	0.75	0.85	1.1
Reported				
$D_{sv} \text{ (m}^2/\text{s)}$	0.030	0.11	0.14	0.23

**SOLUTION**

Using Eq. (3) we find

$$D_{sv} = 12(10)^{0.5} d_t^{0.9} = 38 d_t^{0.9}, \quad [\text{cm}^2/\text{s}]$$

The main problem in using Eq. (12) is choosing a reliable value for the wake fraction  $f_w$ . Hamilton et al. [21] report  $f_w = 1-2.9$  for this range of particle size (mean value of 2), whereas Fig. 5.8 gives  $f_w = 0.32$ . Although not very satisfactory, we average these  $f_w$  values. Thus

$$f_w = \frac{0.32 + 2}{2} = 1.16 \approx 1.2$$

Then Eq. (12) becomes

$$D_{sv} = \frac{(1.2)^2 (0.5) (0.2) (0.06) u_b^2}{3(0.015)} = 0.192 u_b^2$$

Thus we find

$d_t$	0.1	0.3	0.6	1.5 m
$D_{sv}$ , from experiment	0.030	0.11	0.14	0.23 m <sup>2</sup> /s
$D_{sv}$ , from Eq. (3)	0.030	0.08	0.15	0.35 m <sup>2</sup> /s
$D_{sv}$ , from Eq. (12)	0.031	0.11	0.14	0.23 m <sup>2</sup> /s



## Horizontal Movement of Solids

### Experimental Findings

The horizontal movement of solids was first studied by Brötz [22] in a shallow rectangular bed, as shown in Fig. 5. Measuring the rate of approach to uniformity after removal of the dividing plate then gave the information needed to evaluate the horizontal dispersion coefficient  $D_{sh}$ . A similar approach was used by other investigators [23–25].

Heertjes et al. [26] suggested that the wake material scattered into the freeboard by the bursting bubbles could contribute significantly to the horizontal movement of solids. HIRAMA et al. [24] and Shi and Gu [27] used partition plates in the freeboard just above the bed to study this effect.

All of these investigators used rather shallow beds of height between 5 and 35 cm. In contrast, Bellgardt and Werther [28] made measurements in a much larger bed, namely a  $2\text{ m} \times 0.3\text{ m}$  bed about 1 m deep. Quartz sand ( $d_p = 450\text{ }\mu\text{m}$ ) was fluidized, and careful measurements confirmed that vertical mixing was much faster than horizontal mixing, thus justifying the use of a one-dimensional dispersion model in the horizontal direction. They found  $D_{sh} = 6\text{--}25\text{ cm}^2/\text{s}$  for  $u_o = 0.23\text{--}0.73\text{ m/s}$ . Applying their model to coal combustion and gasification, Bellgardt et al. [29] presented a performance model for solid movement that was tested in TVA's 20-MW FBC pilot plant.

Table 1 gives information on the reported studies of horizontal movement of solids. We summarize these findings as follows:

- Comparing the dispersion coefficient for the horizontal movement with the vertical movement of solids (compare Table 1 with Fig. 1), we see that  $D_{sh}$  is roughly an order of magnitude smaller than  $D_{sv}$ .
- $D_{sh}$  increases with bed width. For example, HIRAMA et al. [24] in their very small units found a 60% increase for a doubling in bed width.
- The scattering of solids into the freeboard contributes significantly to  $D_{sh}$  in shallow beds.

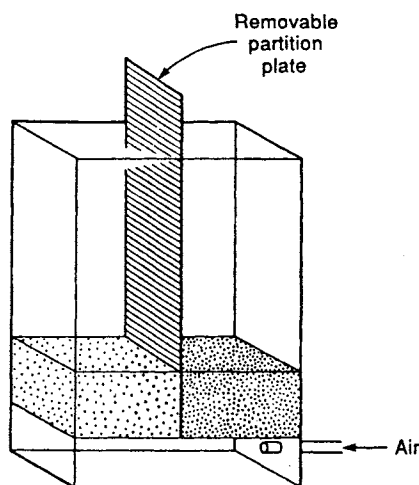


FIGURE 5  
Experimental setup used by Brötz [22] to study the horizontal movement of solids.



TABLE 1 Range of Experimental Data for the Horizontal Movement of Solid

Investigators	Bed (m)	Particles
Mori and Nakamura [23] (1965)	0.9 × 0.3	Polyvinyl chloride
Hirama et al. [24] (1975)	0.4 × (0.042, 0.08, 0.2)	Glass beads, cracking cat.
Bellgardt and Werther [28] (1984)	2.0 × 0.3 $L_f \approx 1$	Quartz sand
Kato et al. [30] (1985)	0.5 × 0.2 Vertical tubes 0.032 Horizontal tubes 0.018	Activated carbon
Shi and Cu [27] (1986)	0.57 × 0.05	Resin, silica gel

- Vertical and horizontal internals reduce  $D_{sh}$  significantly. From Kato et al. [30] we find this effect to be as follows:

	No tubes present	Effective bed diameter with tubes present, $d_{te}$ (cm)				
		15.9	10.6	7	5	~2.6
$D_{sh}$ (cm <sup>2</sup> /s) with vertical tubes	20	—	10	—	6	3
$D_{sh}$ (cm <sup>2</sup> /s) with horizontal tubes	20	8	—	6	—	3

This table shows no appreciable difference between the effect of horizontal and vertical tubes. Kato et al. [30] also found that for their operating conditions  $D_{sh}$  was unaffected by bed height.

### Mechanistic Model Based on the Davidson Bubble

Consider the following mechanism for the horizontal movement of solids in a fine particle bed of fast rising bubbles, as sketched in Fig. 6.

**Postulate.** As a bubble rises, it pushed emulsion aside. However, the solids passing close to the bubble enter its cloud and are then drawn into the wake, whose diameter is roughly  $\alpha$  times the bubble diameter. Solids mix uniformly in the wake and leave the wake from random positions, thereby giving rise to horizontal mixing. Solids further from the bubble move aside as the bubble passes, but return close to their original position.

For this mechanism it is simplest to evaluate the horizontal dispersion coefficient  $D_{sh}$  in terms of the Einstein random walk equation:

$$\begin{aligned}
 D_{sh} &= \frac{(\text{fraction of solids that mix})(\text{mean square distance moved})}{4(\text{time interval considered})} \\
 &= \frac{1}{4} \left( \frac{\text{fraction of bed solids that enter bubble wakes}}{\text{to mix there per unit time}} \right) \overline{\Delta r^2} \quad (13)
 \end{aligned}$$



## Particles

$d_p$ ( $\mu\text{m}$ )	$u_{mf}$ (m/s)	$D_{sh}$ ( $\text{m}^2/\text{s}$ ) at $u_0$ (m/s) and $d_b$ (m)
595	0.295	$1-7 \times 10^{-3}$ at $u_0 = 0.4-0.7$
150, 460	0.024, 0.15	$0.7-2 \times 10^{-3}$ at $u_b d_b = 1-2 \times 10^{-2}$
75	0.0055	$0.6-3 \times 10^{-3}$ at $u_b d_b = 1-6 \times 10^{-2}$
		$d_b = 0.02-0.06$ at $u_0 - u_{mf} = 0.05-0.4$
450	0.17	$0.6-2.5 \times 10^{-3}$ at $u_0 - u_{mf} = 0-0.5$
394-1073	0.035-0.27	$0.1-1 \times 10^{-3}$ at $u_0 - u_{mf} = 0.1-0.6$
		$0.2-1 \times 10^{-3}$ at $u_0 - u_{mf} = 0.15-0.6$
		$d_e = 0.026-0.159$
450, 750	0.076, 0.20	$0.3-0.8 \times 10^{-3}$ at $L_{mf} = 0.02-0.07$
620	0.0905	

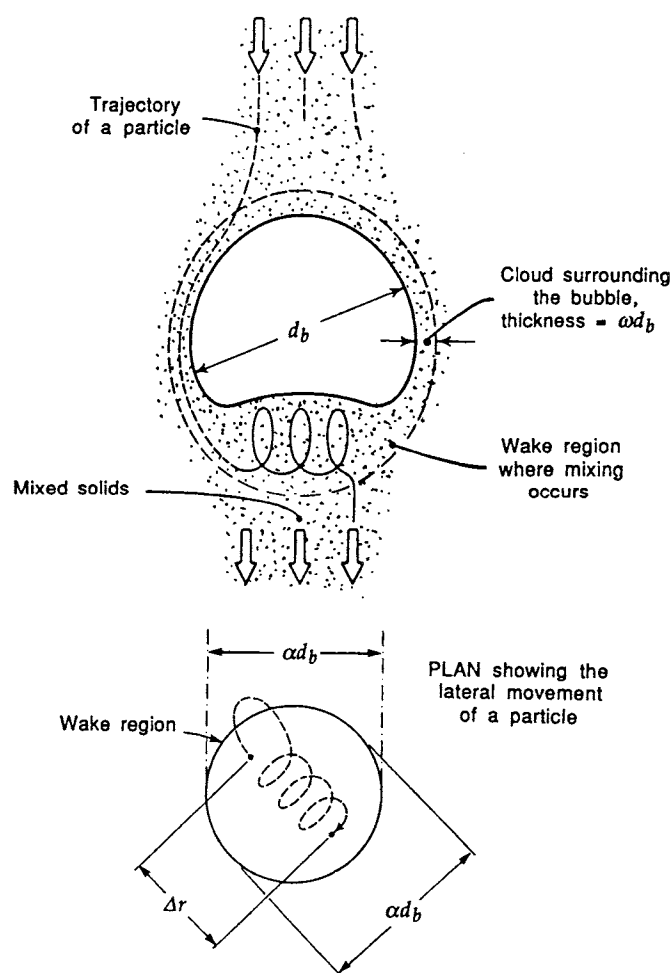


FIGURE 6

The horizontal movement of solids according to the model of Kunii and Levenspiel [31].



Kunii and Levenspiel [31] evaluated the terms of this expression. Thus for clouded Davidson bubbles the term in parentheses depends on bubble size, cloud thickness, and bubble density in the bed. Next, from probability theory the mean square horizontal shift of a particle on passing through the bubble wake is given by

$$\overline{\Delta r^2} = \frac{(\alpha d_b)^2}{4}$$

where  $\alpha d_b$  is the effective diameter of the wake.

Replacing all quantities into Eq. (13) gives, in general, for both fast and intermediate bubbles,

$$D_{sh} = \frac{3}{16} \frac{\delta}{1-\delta} \alpha^2 d_b u_{br} \left[ \left( \frac{u_{br} + 2u_f}{u_{br} - u_f} \right)^{1/3} - 1 \right] \quad (14)$$

For fast bubbles with thin clouds typical of fine particle systems, or  $u_{br} \gg u_f$ , Eq. (14) simplifies to

$$D_{sh} = \frac{3}{16} \frac{\delta}{1-\delta} \frac{\alpha^2 u_{mf} d_b}{\epsilon_{mf}} \quad (15)$$

For fine Geldart A and AB solids ( $d_p = 60$  and  $150 \mu\text{m}$ ), Kunii and Levenspiel found that Eq. (15) with  $\alpha = 1$  fitted their data, while for larger Geldart BD solids (quartz,  $d_p = 450 \mu\text{m}$ ), Bellgardt and Werther [28] found that  $\alpha = 0.77$  well represented their data.

Equations (14) and (15) do not account for the scattering of solids at the bed surface, and Shi and Gu [27] presented a model that does account for it. However, since this freeboard scattering is restricted to the top layer of bed solids, its effect on  $D_{sh}$  for the bed as a whole becomes smaller for deeper beds.

Equation (15) can be used for approximate prediction of  $D_{sh}$  for deep beds, but more importantly it suggests how changes in system variables such as  $u_{mf}$ ,  $u_o$ , and  $d_b$  will affect  $D_{sh}$ . Example 2 concerns  $D_{sh}$ .

## EXAMPLE 2

Horizontal  
Drift of  
Solids

Bellgardt and Werther [28] presented the following data on the horizontal dispersion coefficient for quartz solids in fluidized beds. Compare the predicted  $D_{sh}$  from Eq. (14) or (15) with their findings.

### Data

Bed size:  $0.3 \times 2 \text{ m}$ ,  $L_{mf} = 0.83 \text{ m}$

Quartz sand:  $d_p = 450 \mu\text{m}$ ,  $\epsilon_{mf} = 0.42$ ,  $u_{mf} = 0.17 \text{ m/s}$

$u_o \text{ (m/s)}$	0.37	0.47	0.57	0.67
Reported $D_{sh} \text{ (m}^2\text{/s)}$	0.0012	0.0018	0.0021	0.0025

From Fig. 6.10(a) we estimate  $d_b = 0.10\text{--}0.14 \text{ m}$  for this range of gas velocities.

### SOLUTION

Let us show the solution for the first data point,  $u_o = 0.37 \text{ m/s}$ , and for the smallest estimated bubble size,  $d_b = 0.10 \text{ m}$ . Then the first problem is to decide whether to



use Eq. (14) or (15). For this determine whether  $u_{br} \gg u_f$ . Now

$$u_{br} = 0.711(d_b g)^{1/2} = 0.711(0.1 \times 9.8)^{1/2} = 0.70 \text{ m/s}$$

$$u_b = u_o - u_{mf} + u_{br} = 0.37 - 0.17 + 0.70 = 0.90 \text{ m/s}$$

$$u_f = \frac{u_{mf}}{\epsilon_{mf}} = \frac{0.17}{0.42} = 0.40 \text{ m/s}$$

Since we do not have  $u_b \gg u_f$ , we use Eq. (14) instead of Eq. (15) to calculate  $D_{sh}$ . In addition, since we are not dealing with fine Geldart A or AB solids, we take  $\alpha = 0.77$  in this equation (see just after Eq. (15)).

The last quantity needed before using Eq. (14) is  $\delta$ . Since  $u_{br}$  is close to  $u_f$ , we use Eq. (6.27) to give

$$\delta \cong \frac{u_o - u_{mf}}{u_b + u_{mf}} = \frac{0.37 - 0.17}{0.90 + 0.17} = 0.187$$

Substituting all into Eq. (14) gives

$$D_{sh} = \frac{3}{16} \frac{0.187}{1 - 0.187} (0.77)^2 (0.1)(0.70) \left[ \left( \frac{0.70 + 2 \times 0.40}{0.70 - 0.40} \right)^{1/3} - 1 \right] \\ = 0.0013 \text{ m}^2/\text{s}$$

Similar calculations are made for other  $u_o$  values. The final results and comparison with the experimental values gives

$u_o$	0.37	0.47	0.57	0.67 m/s
$D_{sv}$ calculated { at $d_b = 0.10 \text{ m}$	1.3	1.9	2.5	$3.2 \times 10^{-3} \text{ m}^2/\text{s}$
at $d_b = 0.14 \text{ m}$	1.4	2.2	2.9	$3.6 \times 10^{-3} \text{ m}^2/\text{s}$
$D_{sv}$ from experiment	1.2	1.8	2.1	$2.5 \times 10^{-3} \text{ m}^2/\text{s}$

The calculated values are somewhat high.

## Segregation of Particles

Numerous processes require fluidizing a mixture of solids of very different density. As an example, in one step in the production of titanium or zirconium a mixture of the metal oxide (high-density solid) and coke (very low density) is fluidized by chlorine gas at high temperature. To achieve close to 100% conversion of chlorine, bed bubbling should not be too vigorous; hence the gas velocity should not be too high. On the other hand, the gas velocity should not be too low or solids will separate out. What size ratio of solids and what gas velocity should be used in such situations? The whole question of the mixing-segregation equilibrium is of vital concern in these situations.

### Mixing-Segregation Equilibrium

Much has been reported in recent years on the mixing-segregation phenomenon in gas fluidized beds, especially on binary systems of different size and/or density; see [32]. Here, particle segregation occurs at close to  $u_{mf}$  of the biggest or heaviest particles in the bed. Also, this whole question mainly concerns large particle systems.

Cooperative investigations on particle segregation were carried out by Rowe et al. [33,34], Nienow et al. [35,36], Chiba [37-39] and others, in which the following special terminology was used for the fluidized bed components:

- jetsam*: component that ultimately sinks
- flotsam*: component that floats to the top of the bed



We summarize those findings as follows:

With solids of the same size but different density, the bed segregates readily. When this occurs, the dense material forms a relatively pure bottom layer. The upper layer always contains some of the denser solids, more or less uniformly dispersed.

Particles of different size but the same density will also segregate, but not easily. Even particles an order of magnitude different in diameter will mix fairly uniformly at moderate bubbling conditions. With a wide size distribution of particles rather than a sharp cut of two distinct sizes, we may expect much less segregation.

When the gas velocity is close to  $u_{mf}$ , the segregation of the jetsam can be severe. At higher gas velocities it is less severe. Figure 7 illustrates the segregation pattern in binary mixtures; Fig. 8 illustrates this behavior for mixtures of commercial powders. Thus, Fig. 8(a) shows a sharp segregation of denser material (density ratio  $\approx 1.8:1$ ), and Fig. 8(b) shows that even with very different particle sizes (size ratio = 3.5:1) segregation is minor. Both systems display less segregation when the gas velocity is raised.

Rowe et al. [33] proposed using a solids mixing index defined as

$$M = \left( \frac{\text{fraction of jetsam in the top portion of the bed}}{\text{fraction in a well-mixed bed}} \right) = \frac{X_{sJ, \text{ top}}}{\bar{X}_{sJ}} \quad (16)$$

$M = 0$  and 1 correspond to complete segregation and complete mixing, respectively.

Noting that the jetsam fraction in Fig. 8 is practically constant in a large portion of the bed, we can use this value to get an approximation for  $M$ ; thus

$$M \cong \frac{X_{sJ, \text{ straight-line portion}}}{\bar{X}_{sJ}} \quad (17)$$

Factors affecting particle segregation were studied by Rowe and Nienow [34,35] in terms of this index.

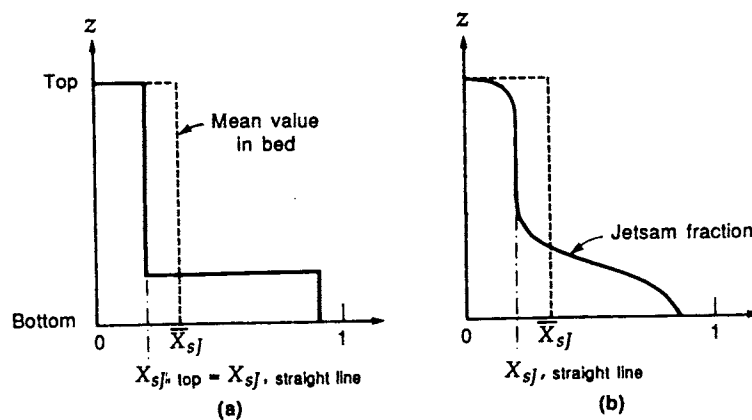


FIGURE 7

Distribution of solids in strongly segregating binary mixtures: (a) idealized segregation at low fluidizing velocity; (b) pattern of segregation at vigorous bubbling conditions.



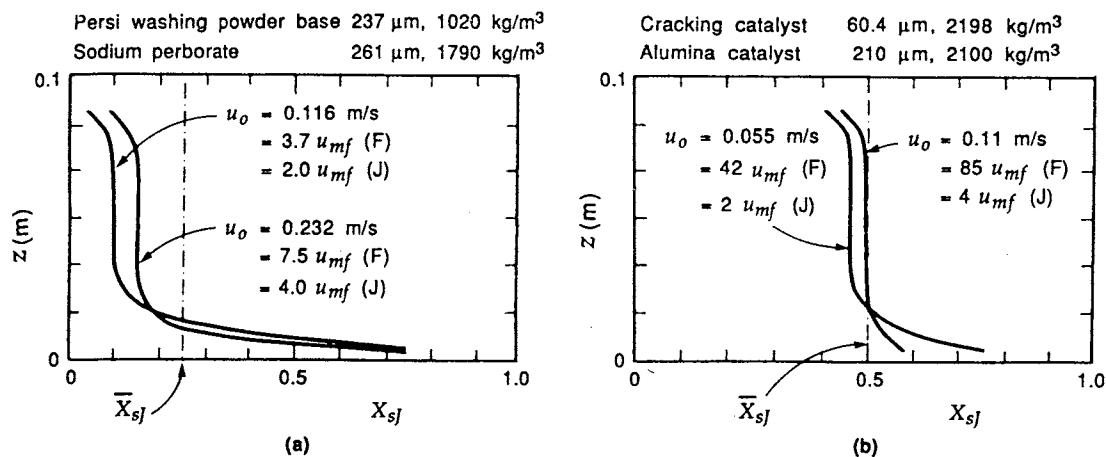


FIGURE 8

Vertical segregation of commercial solids;  $d_t = 0.141 \text{ m}$ ,  $L_m = 0.1\text{--}0.15 \text{ m}$ ; adapted from Rowe et al [33]: (a) different density materials; (b) different sized materials.

Rowe et al. [33] were the first to suggest that rising bubbles are the vehicle for particle segregation. Thus, all solids, both flotsam and jetsam, are carried up the bed in the bubble wakes. However, only the larger denser particles preferentially move down the bed as a bubble passes by. They emphasized that this upflow in the bubble wake is the only way that the smaller less dense particles can reach the top of the bed.

Mathematical models to account for the axial distribution of solids at a stable mixing-segregation equilibrium have been proposed by several research groups. Chiba et al. [39] reviewed and summarized these works. They also explained the preferential downflow of denser solids as follows: the denser larger particles tend to fall preferentially through the temporarily disturbed region (the wake) behind the bubbles. This downward movement of jetsam is clearly analogous to the jiggling separation technique.

Chiba et al. [38] found that an increase in pressure promoted solid mixing, and explained this in terms of increased wake fractions. This observation suggests that for their system the mixing of bed solids by the upflow of wakes was more important than the preferential downflow of jetsam from the wakes.

### Steady State Separation of Particles

A number of operations are being explored that require fluidization with separation of two different kinds of solids: for example, driers in which fine wet solids are mixed with a hot solid heat carrier, and some advanced combustor-gasifiers that fluidize dolomite and char. Some researchers [35,40–43] have studied these systems.

Note that with one inflow and one outflow stream for a mixture of solids that readily segregate, the bed composition will adjust itself so that at steady state the outflow composition will equal the inflow. Of course, the location of the outflow will influence the bed composition. Figure 9 illustrates this. With two outflow streams, one should be able to get a good separation of components, and Chiba et al. [36] showed how to predict the composition of the two outflow